A review of Bose-Einstein condensation
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Abstract

This is a review on Bose-Einstein condensation. In the review a historical perspective will be presented, along with some rudimentary theory, experiments and possible applications.

The theory will mainly treat the effect of spin-nucleus interaction, but also the effect of a magnetic field, which is crucial in the production of most Bose-Einstein condensates. A model for calculating the critical temperature is also provided.

The experimental section contains methods of trapping and cooling atoms to the low temperatures required. The same principles, but with refined technique, are applied today to reduce the temperature to a few mK or even lower.

Finally, the Bose-Einstein condensates can be used to study fundamental properties of atoms and possibly be used in a similar way as a laser.

Contents

1 Introduction

The recent observation of Bose-Einstein condensation (BEC) in dilute atomic gases is quite possibly the most exciting development in atomic, molecular, and optical physics since the invention of the laser. It gives us with a great opportunity to study coherent matter, because the relatively weak particle interactions make the theory tractable, and powerful experimental techniques provide detailed measurements of the condensate properties.

Fermionic atoms obey the Pauli exclusion principle while bosonic atoms does not have this property. This means that bosonic atoms all can be in the same quantum state. This is called a Bose-Einstein condensate and was theoretically predicted 1925 by Albert Einstein [1]. The BEC can only occur in non-interacting gas-clouds at extremely low temperatures because the interactions would ruin the homogenity. As the temperature goes to zero the momentum spread decreases, and thus the uncertanity in position increases. This can be viewed as the atoms spread over each other and occupy the same state.

A BEC, ^{4}He, was experiementally found 1932 by Kaptiza [2]. Below 2.19 K, the λ point, the viscosity of liquid helium dropped drastically which allowed the helium to flow through holes with diameters as small as 100 Å. However, it wasn't until 1998 that it was directly proven that ^{4}He was actually a BEC [3].

In the beginning of the 1970s it was discovered that even fermionic atoms can become a BEC, in a similar way as electrons makes Cooper pairs in super conductors. The effect was that ^{3}He, which is a fermion, became superfluid and the discovery lead to the Nobel prize in Physics 1996 for David M. Lee, Douglas D. Osheroff and Robert C. Richardson.

Recently, the interest in BECs has increased with the discovery of several new BECs, like hydrogen, sodium and rubidium. This has allowed a more extensive study of the properties of BEC, and also spawned the hope of creating an "atom-laser".

2 Theory

2.1 The transition temperature

The total number of particles in terms of the critical temperature T_c, for a density of states $g(x)$ and Bose distribution $f^0(x)$ with chemical potential

$\mu = 0$ and temperature $T = T_c$, is generally given by

$$N = \int_0^\infty dx g(x) f^0(x) = C_\alpha (kT_c)^\alpha \int_0^\infty dx \frac{x^{\alpha-1}}{e^x - 1}, \tag{1}$$

where $x = \epsilon/kT_c$ and C_α and α are constants depening on the potential used for the confinement of the atoms. They can be obtained by solving the Schrödinger equation for that potential. For a uniform Bose gas contained in a box of volume V and particle density n the integral can be calculated numerically and the result gives the critical temperature. For ^{4}He this is

$$T_c = 3.3 \frac{\hbar^2 n^{2/3}}{mk} \approx 2.17, \tag{2}$$

which is in agreement with the value obtained in experiments.

2.2 Atomic structure

The splitting of the energy levels in a BEC are due to the hyperfine interaction, which has the Hamiltonian

$$H_{hf} = A\mathbf{I}.\mathbf{J}, \tag{3}$$

where A is a constant, $\mathbf{J}$ is the total angular momentum electron and $\mathbf{I}$ is the total angular momentum of the nucleus. The total angular momentum is given by

$$\mathbf{F} = \mathbf{I} + \mathbf{J}, \tag{4}$$

and this can be used to express $\mathbf{I}.\mathbf{J}$ in terms of the quantum numbers I, J, F

$$\mathbf{I}.\mathbf{J} = \frac{1}{2}(F(F+1) - I(I+1) - J(J+1)). \tag{5}$$

For example, for rubidium, we have $E_1 = -5A/4$, $E_2 = 3A/4$ and thus the splitting

$$\Delta E = h\Delta\nu = 2A. \tag{6}$$

The Zeeman effect with the Hamiltonian

$$H_{tot} = A\mathbf{I}.\mathbf{J} + CJ_z + DI_z, \tag{7}$$

gives eigenvalues with either increasing, or decreasing energy with respect to increasing magnetic field.

3 Experiment - Trapping and cooling

This field is now in its infancy, and there are many open questions about condensate thermodynamic properties, collective and vortex excitations, the damping and (possibly nonlinear) couplings of these excitations, critical properties, optical properties, role of atomic interactions, stability with respect to recombination, behavior of negative scattering length condensates, finite size effects, and the role of internal degrees of freedom. Other fascinating questions concern the basic phenomenon itself: the formation of a coherent state of matter. The conditions under which the phase of this matter can be observed, and the extent to which it may vary in space or time, are of great interest.

3.1 Magnetic quadrupolar trapping

If atoms move sufficiently slowly in a magnetic field they will remain in the same quantum state. The motion is then said to be adiabatic. With the quadrupolar magnetic field

$$\mathbf{B} = B'(x, y, -2z) \tag{8}$$

some atoms will minimize their energy where the field is lowest others where the field is highest (cf. Atomic structure). The atoms that minimize their energy where the field is lowest will stay in the trap while the others will be repelled. However, there is a problem with this setup. When the approaches the node of the field the energy separation decreases and the motion isn't adiabatic anymore. This means that even small perturbations can induce a transition between substates. The perturbation can be as small as the time-dependance of the magnetic field due to the motion of the atom. Thus, a "low-seeking" atom can make a transition to a "high-seeking" and hence be repelled. Eventually, all of the atoms will have been repelled. This problem can be solved in different ways, e.g. with having a laser going thorugh the node. It can also be solved by superimposing a rotating, but spatially uniform magnetic field of constant amplitude. In principly, our magentic field would look like:

$$\mathbf{B} = \mathbf{B}(\vec{r}) + \mathbf{B}(t) = (B'x + B_0 \cos \omega t, B'y + B_0 \sin \omega t, -2B'z). \tag{9}$$

Hence, the effect of the time-depending field will be to move the momentary node thus avoiding that all the slow atoms disappear. At small distances from the original node, the time-averaged field will be

$$\langle B \rangle_t \approx B_0 + \frac{B'^2}{4B_0}(x^2 + y^2 + 8z^2), \tag{10}$$

which never vanishes, thus avoiding the "hole". However, we also add $E \approx -\mu\langle B\rangle_t$ to the energy of the atom.

3.2 Laser cooling

Suppose that we have two opposed lasers with the same intensity and the same frequency ω with ω just below the transition energy ω_{eg} between the ground state and the first excited state. This gives us the line width of the photon absorption as

$$\Gamma = \frac{1}{\tau} = CL(\omega), \tag{11}$$

with $C = C(\vec{r}, t)$ a constant with respect to ω and

$$L(\omega) = \frac{\Gamma_e/2\pi}{(\omega - \omega_{eg})^2 + (\Gamma_e/2)^2}. \tag{12}$$

An atom that isn't moving will recieve as many photons from the right as from the left, thus giving no net result. The Doppler shift means that an atom which is moving right with a velocity v will change the effective frequency to $\omega \pm vk$, where k is the wavenumber of the photon. The right moving photons will have a reduced absorption and the left moving will have an increased absorption, thus working as a "break" for the atom[1].

The advantages with laser cooling in comparison with magnetic cooling are

1. researchers now have the chance to create BECs of atoms that don't respond to magnetic fields

2. a laser beam can control atoms to a high degree, for example by guiding them down a hollow optical fiber

4 Applications

4.1 Atom lasers

Perhaps the most exciting prospect is that processes related to Bose condensation might allow us to produce coherent matter wave sources, or "atom lasers." That is, we may be able to build a device that would put out a bright, monochromatic, diffraction-limited matter wave, with a coherence

[1] An excellent grapical interpretation of the cooling of an atom is given at *http://www.colorado.edu/physics/2000/bec/index.html*

length much larger than the size of the source. An atom laser might find powerful applications in areas such as atom interferometry, atomic holography (i.e. the direct formation of a desired material object by coherent manipulation of the matter waves), or in other as yet unimagined areas. Various proposals for atom lasers include coherent coupling of atoms out of a condensate with simultaneous "pumping" of the condensate, and optical cycling of cold atoms into "lasing" atomic modes.

References

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[6] *http://amo.phy.gasou.edu/bec.html/popular.html#press*