

# Gravitational Waves

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## **Abstract**

A review on the properties of gravitational waves is presented along with comparisons with electromagnetic theory. Gravitational waves behave very much like light waves, except for that they are second rank tensors instead of being vectors, like light-waves. The different sources of gravitational radiation are discussed, and their origins and strengths. Finally, the development on the experimental front is treated. This is done mainly on the two experiments LISA and ALLEGRO representing the interferometer and the resonance detector types, respectively.

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## 1 Introduction

Ever since the introduction of Einstein's general theory of relativity, gravitational waves has been of interest. They arise quite naturally from the theory and were conceived by Einstein himself early after the general relativity. They are, if not an inevitable, but at least a very plausible part of the theory. If they were not to exist, we would have to reconsider Einstein's equations and try to explain why this would be so. This could be an exciting subject for further studies in the area.

Gravitational waves are a natural part of general relativity and astrophysics as they mainly stem from huge gravitational objects rapidly changing form, like supernovae explosions, binary stars revolving about each other, masses falling into black holes and so on.

In 1993, Taylor and Hulse were awarded with the Nobel prize for the discovery of indirect evidence of the existence of gravitational waves. A neutron star loses an amount of rotational energy which currently can only be conceived of as being caused by gravitational radiation. Their measurements of radio waves from a binary pulsar designated PSR1913+16 show that the pulsar's 8-hour orbit around the neutron star is gradually contracting; the faster the pulsar revolves around the neutron star, the smaller its orbit gets. As the rate of decrease agrees to within 0.5 percent with predictions derived from the general theory of relativity, the finding is excellent circumstantial evidence for orbital decay being a result of energy lost by gravitational radiation. Even though the gravitational radiation itself was not detected, Taylor and Hulse shared the 1993 Nobel Prize in Physics for this work.

In this brief review on the subject, firstly some general properties of gravitational waves will be presented. This is done as to acquaint the reader with the concept, and to show the many similarities with electromagnetism, thus facilitating the comprehension of the phenomenon. The second section treats the sources for gravitational radiation without any deeper theoretical argumentation. The principles can be seen easily enough from the end of the first section. Two types of sources for gravitational waves are discussed, periodical and catastrophic. The first being more permanent, the second more intensive. Gravitational waves are currently a hot issue and several attempts of detection is being devised and put into action. There are mainly two types of gravitational wave's detectors, one technique from the 60's, resonance monitoring, and a second one, more modern, interferometric method. Primarily the next generation of detectors are discussed, where more sophisticated methods for detection are used.

## 2 Theory

### 2.1 Introduction

The theory of gravitational waves was originally developed by Albert Einstein 1916 as a part of his extensive study on gravitation. The theory of gravitational radiation resembles a lot the theory of electromagnetic theory. Throughout this section comparisons will be made to show the similarities. When visualizing what really happens it is often helpful to think about the gravitational wave as an electromagnetic wave and mass as the charge affected by the field. The reader is supposed to be familiar with tensor analysis and somewhat with general, or at least special, relativity. If not, *Basic Relativity* by Mould gives a good introduction to these subjects. We will denote

$$x_\mu = (t/c, x, y, z) \quad (1)$$

and

$$x_\alpha x^\alpha = -(t/c)^2 + x^2 + y^2 + z^2 \quad (2)$$

and

$$\partial_\mu = \frac{\partial}{\partial x^\mu}. \quad (3)$$

Normally, indices  $\alpha$  and  $\beta$  will be used as dummy indices, while  $\mu$  and  $\nu$  will be used for vectors and tensors.

### 2.2 Polarization

If we neglect the curvature of general relativity<sup>1</sup> the metric of space-time be written on the form:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \quad (4)$$

where  $h_{\mu\nu}$  is a small perturbation in space-time caused by gravitational waves and  $\tilde{g}_{\mu\nu}$  is the normal Minkowski metric tensor:

$$\tilde{g}_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (5)$$

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<sup>1</sup>This is called a linearized theory of gravity in vacuum. The universe is supposed to be a flat, empty space-time, at least from a local viewpoint. This approximation only holds for domains with weak gravitational forces.

Starting from the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}, \quad (6)$$

where  $R = g^{\mu\nu}R_{\mu\nu}$  is the contraction of the Ricci tensor  $R_{\mu\nu}$ ,  $G$  is the universal constant of gravitation, and  $c$  is the speed of light in vacuum. The 16 terms of  $R_{ij}$  is reduced to ten with the symmetry condition  $R_{ij} = R_{ji}$  we have. In the linearized theory of gravity the Ricci tensor can be written in terms of the metric as

$$R_{\mu\nu} = \frac{1}{2}(\partial_\alpha\partial_\nu h_\mu^\alpha + \partial_\mu\partial_\alpha h_\nu^\alpha - \partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu\tilde{g}^{\alpha\beta}h_{\alpha\beta}), \quad (7)$$

which gives the Einstein's equations:

$$\begin{aligned} \partial^\alpha\partial_\nu h_{\mu\alpha} + \partial^\alpha\partial_\mu h_{\nu\alpha} - \partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial_\nu\tilde{g}^{\alpha\beta}h_{\alpha\beta} - \tilde{g}_{\mu\nu}(\partial^\alpha\partial^\beta h_{\alpha\beta} - \\ \partial_\alpha\partial^\alpha\tilde{g}^{\alpha\beta}h_{\alpha\beta}) = \frac{16\pi G}{c^4}T_{\mu\nu}. \end{aligned} \quad (8)$$

We make a transformation:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}h_{\alpha\beta} \quad (9)$$

and the Einstein's equations takes simpler form:

$$\partial_\alpha\partial^\alpha\bar{h}_{\mu\nu} - \tilde{g}_{\mu\nu}\partial^\alpha\partial^\beta\bar{h}_{\alpha\beta} + \partial_\nu\partial^\alpha h_{\mu\alpha} + \partial_\mu\partial^\alpha h_{\nu\alpha} = \frac{16\pi G}{c^4}T_{\mu\nu}. \quad (10)$$

We can impose the guage condition:

$$\partial^\alpha h_{\mu\alpha} = \partial_\alpha h^{\mu\alpha} = 0 \quad (11)$$

without loss of generality. This can be compared with the guage condition in electromagnetic theory  $\partial^\alpha A_\alpha = 0$ . The guage condition in gravitational theory eliminates all but the first term in equation (10) leaving us with the equation:

$$\partial_\alpha\partial^\alpha\bar{h}_{\mu\nu} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar{h}_{\mu\nu} = \square\bar{h}_{\mu\nu} = \frac{16\pi G}{c^4}T_{\mu\nu} \quad (12)$$

which can be compared with the equivalent in electromagnetism  $\square A_\mu = \mu_0 J_\mu$ . With the four guage-conditions, the ten field equations are reduced to only six.

In empty space-time, the energy-momentum tensor of matter is zero,  $T_{\mu\nu} = 0$ . The Einstein's equations then has the form:

$$\square\bar{h}_{\mu\nu} = 0, \quad (13)$$

which is a partial differential equation of second order with the solution

$$\bar{h}_{\mu\nu} = \Re[\bar{h}_{\mu\nu}^0 e^{ik_\alpha x^\alpha}], \quad k_\alpha k^\alpha = 0, \quad (14)$$

where  $\bar{h}_{\mu\nu}^0$  is a constant tensor determining the orientation of the wave in space. Once again, the analogy with electromagnetism is obvious with  $\bar{h}_{\mu\nu}$  is replaced with  $A_\mu$ .

Combining equation (11) with equation ( $k_\alpha k^\alpha = 0$ ) the guage condition can now be written as

$$\bar{h}_{\mu\alpha}^0 k^\alpha = 0, \quad (15)$$

which means that in the direction of the wave, the field is zero, or in other words, the gravitational waves are transversal. Even with the guage condition there is still a freedom in the equations, since the coordinate transformation

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + v_{\mu\nu} + v_{\nu\mu} \quad (16)$$

with  $\square v_\mu = 0$  leaves the coordinate system unchanged. This can be used to further reduce the number of equations, arriving at only two left. These conditions can be arranged in such a manner that:

$$\bar{h}_{\mu\nu} = 0, \quad \bar{h}_{\alpha\alpha} = 0, \quad (17)$$

which also can be written as

$$h_{\mu 0} = 0, \quad h_{\mu\mu} = 0. \quad (18)$$

This means that the time-components are all zero and that  $h_{\mu\nu}$  is trace-less. With this choice of coordinates, we have defined the Transversal-Traceless (TT) guage. If we let the wave propagate in the x-direction,  $k_\mu = (k, k, 0, 0)$  the TT-guage along with equation (15) and symmetry gives

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{yy} & h_{yz} \\ 0 & 0 & h_{yz} & -h_{yy} \end{bmatrix} \quad (19)$$

which can be decomposed to

$$h_{\mu\nu} = h_{yy} \mathbf{e}_+ + h_{yz} \mathbf{e}_\times, \quad (20)$$

where

$$\mathbf{e}_+ = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \mathbf{e}_\times = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (21)$$

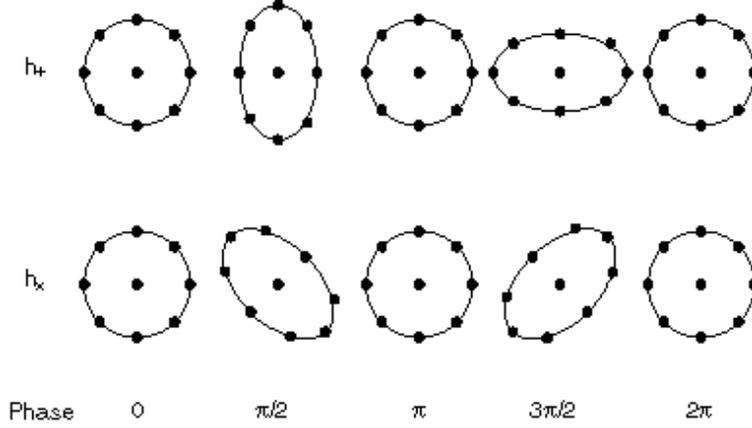


Figure 1: Deformation of a test ring of matter produced by linearly polarized waves.

Hence, the wave equation can be split up in two components

$$\mathbf{h}_+ = \Re \left[ \mathbf{h}_+^0 e^{ik(\times-t/c)} \right], \quad \mathbf{h}_\times = \Re \left[ \mathbf{h}_\times^0 e^{ik(\times-t/c)} \right] \quad (22)$$

which are the two polarization states of gravitational radiation. Let us see what the two states really mean. In  $\mathbf{e}_+$  the y-direction decreases when the z-direction increases and vice versa, see figure 2.2. To visualize  $\mathbf{e}_x$  we rotate the coordinate system 45 degrees where it takes the same form as  $\mathbf{e}_+$  in the normal coordinate system. Thus, it represents the same kind of oscillation but rotated 45 degrees with respect to  $\mathbf{e}_+$ .

With a polarized state  $\mathbf{h}_+$ , the metric of space-time would become

$$g_{\mu\nu} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1+h & 0 \\ 0 & 0 & 0 & -1-h \end{bmatrix} \quad (23)$$

This gives the strain in a material as

$$\epsilon = \frac{\delta l}{l} = \frac{1}{2}h \quad (24)$$

This is a summary of what we have seen in the previous section, to better see the similarities.

Equation	Gravitation	Electromagnetism
Gauge condition	$\partial^\alpha \bar{h}_{\alpha\nu} = 0$	$\partial^\alpha A_\alpha = 0$
Wave equation	$\square \bar{h}_{\mu\nu} = \frac{16\pi G}{c^4} T_{\mu\nu}$	$\square A_\mu = \mu_0 J_\mu$
Wave equation in vacuum	$\square \bar{h}_{\mu\nu} = 0$	$\square A_\mu = 0$
Plane waves in vacuum	$\bar{h}_{\mu\nu} = \Re[\bar{h}_{\mu\nu}^0 e^{ik_\alpha x^\alpha}]$	$A_\mu = \Re[A_\mu^0 e^{ik_\alpha x^\alpha}]$

We have also seen that gravitational waves only have two polarized states which are both transversal, that is, perpendicular to the direction of propagation.

### 2.3 Electric Quadrupole Radiation

In electromagnetic theory the electric dipole radiation is given by

$$L_{ed} = \frac{2}{3} \ddot{\mathbf{d}}_e^2, \quad (25)$$

$$\mathbf{d}_e = \sum_i q_i \mathbf{x}_i \quad (26)$$

where  $L$  is the luminosity,  $d_e$  is the electric dipole moment and  $q_i$  is the charge of particle  $i$ . The equivalent in the theory of gravity is obtained by replacing the charge,  $q_i$  with the mass,  $m_i$ :

$$L_{gd} = \frac{2}{3} \ddot{\mathbf{d}}_g^2, \quad (27)$$

$$\mathbf{d}_g = \sum_i m_i \mathbf{x}_i \quad (28)$$

However,  $L_{gd} = 0$  by conservation of momentum:

$$\dot{\mathbf{d}} = \sum_i m_i \dot{\mathbf{x}}_i = \sum_i \mathbf{p}_i \equiv 0 \Rightarrow L_{gd} = 0. \quad (29)$$

That is, there is no gravitational dipole radiation.

The second strongest type of radiation in electromagnetic theory is the magnetic dipole radiation which is given by the second derivative of the magnetic dipole moment:

$$\boldsymbol{\mu}_m = \sum_i \mathbf{r}_i \times \mathbf{J} = \sum_i \mathbf{r}_i \times (q\mathbf{v}), \quad (30)$$

which would give a gravitational "magnetic" dipole moment of

$$\boldsymbol{\mu}_g = \sum_i \mathbf{r}_i \times \mathbf{J} = \sum_i \mathbf{r}_i \times (m\mathbf{v}) = \sum_i \mathbf{J}_i = 0, \quad (31)$$

where  $\mathbf{J}$  is the angular momentum. Hence, conservation of angular momentum gives that there is no dipole radiation for gravitation what so ever.

The first type of radiation that can occur is from the quadrupole moment, and the gravitational luminosity can be expressed as:

$$\dot{\epsilon} = -\frac{C}{5c^5} \ddot{\ddot{D}}_{ij}, \quad i, j = 1, 2, 3 \quad (32)$$

where  $D_{ij}$  is the mass-energy quadrupole tensor of the source:

$$D_{ij} = \int \rho \cdot (x_i x_j - \frac{1}{3} \delta_{ij} |x|^2) dV, \quad i, j = 1, 2, 3. \quad (33)$$

## 2.4 Summary

Gravitational waves, as predicted through general relativity has a number of properties simliar to those of electromagenitic radiation:

1. The velocity of the gravitaional waves is the speed of light,  $v = c$ .
2. Gravitational radiation has two different polarized states.  
Gravitational radiation has only transversal components.
3. Gravitational waves are emitted through quadrupole radiation.

### 3 Sources of Gravitational Waves

There are two types of sources of considerable gravitational waves, periodical and catastrophic. Examples of periodical sources are spinning rods, spinning stars, spinning of asymmetric black holes and double stars. Catastrophic events can be the explosion of a supernova or a strong gravitational collapse. Generally, the energy emitted from the catastrophic events is much greater than that from the periodic sources. However, the very fact that they are periodical makes it relatively easy to monitor them and fine tune our detectors to reduce the noise of the system. Let us start with investigating a couple of periodical sources of gravitational waves.

#### 3.1 Spinning Rod

The energy radiated from a system is principally given by equation (32). To show the order of magnitude of gravitational waves, we will illustrate the power output from a steel rod of 20 meters length and 1 meter radius spinning as fast as permitted without breaking. The power output for a rotating solid cylinder is given by:

$$\dot{\epsilon} = -\frac{32}{5} \frac{G}{c^5} I^2 \omega^6, \quad (34)$$

where  $I$  is the moment of inertia about the spin-axis and  $\omega$  is the angular velocity of the object. For the steel rod, the result is very insignificant:

$$\det \epsilon = -2.2 \times 10^{-29} J/s. \quad (35)$$

This tiny energy is then to be distributed among all the particles that it passes which gives a small effect indeed. Of course this was to be expected as we haven't seen any direct evidence of gravitational waves, which we would have observed had the effect been greater. Hence, laboratory-created gravitational waves will not be detected yet for many a year. As we cannot deduce the existence of gravitational waves through a laboratory let us turn to other (natural) sources of gravitational waves.

#### 3.2 Spinning Star

As previously stated, it is an asymmetry around the axis of revolution that gives rise to the quadrupole gravitational radiation. Hence, it is only emitted by non-spherical stars<sup>2</sup> with an asymmetry around the axis of rotation. The asymmetry,  $e$  gives a power output of about

$$\dot{\epsilon} = -\frac{288G}{45c^5} I^2 \omega^6 e^2. \quad (36)$$

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<sup>2</sup>Star means a star in any state of evolution, from a young star to a neutron star and a black hole.

The asymmetry is  $e = (a - b)/\sqrt{ab}$ , where  $a$  and  $b$  are the principal axis in the equatorial plane. The highest power output will certainly occur for stars with a low radius because this will make the star rotate even faster, by conservation of angular momentum, thus increasing  $\omega$  and consequently the power output. This fact makes dense objects like neutron stars and black holes a primary target for detectors. However, our scope of measurement still lies several orders of magnitude above the calculated power output.

### 3.3 Double-Star System

The power output for two objects<sup>3</sup> of masses  $m_1$  and  $m_2$  rotating about each other with an angular velocity of  $\omega$  and a distance between mass-centers of  $r$  is

$$\dot{\epsilon} = -\frac{32G}{5c^5} \left( \frac{m_1 m_2}{m_+ m_2} \right)^2 r^4 \omega^6. \quad (37)$$

Once again, the angular velocity  $\omega$  depends on the radius of rotation, thus conserving angular momentum. Hence, the closer the two objects are to each other, the higher will the power output (obviously, they will eventually crash making the formula invalid).

Even if the power output is rather large, it is greatly reduced by the distance between us and all double-star systems of significance. For example, the binary  $\beta$ Per has a radiated energy of about

$$\dot{\epsilon} \approx 10^{21} \text{ J/s} \quad (38)$$

which gives

$$\dot{\epsilon}/A \approx 10^{-20} \text{ J/s} \quad (39)$$

here on earth, which is still a very small value.

About fifty percent of all stars in the universe are belonging to a multiple system. It is estimated [1] that there should be more than  $10^6$  double stars in our galaxies with collapsed bodies, which are more interesting due to their higher angular velocity.

### 3.4 Body Falling into a Black Hole

Let us now move on to some catastrophic events, which on one hand gives more energy, but on the other hand are rather short-lived. For an object falling into a collapsed body of mass  $M$ , the gravitational energy emitted is

$$\epsilon = 0.0025 m^2 c^2 / M, \quad (40)$$

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<sup>3</sup>Objects in this context means a star in any state of evolution, from a young star to a neutron star and a black hole. However, the formula would also be valid for any type of massive objects.

where  $m$  is the mass of the object falling into the collapsed body. This energy is peaked around the frequency

$$\nu = 4.9 \times 10^3 M_\odot / M \text{ Hz.} \quad (41)$$

For example, a neutron star with mass  $m = 2M_\odot$  falling into a black hole of mass  $M = 10M_\odot$  would produce an energy of roughly  $10^{44}$  Joules with a peak-frequency of about 6 kHz. If this burst would occur in the relative proximity of earth ( $\lesssim 1000\text{pc}$ ) within a millisecond, it would be possible to observe it with current detectors.

### 3.5 Gravitational Collapse

Objects that collapse symmetrically give no gravitational waves. However, the star is rotating while collapsing and bursts of gravitational waves may be emitted during the process. The total energy emitted will depend on the particulars of the collapse, but its spectrum is expected to be continuous from zero to the critical frequency

$$\nu = \sqrt{\pi G \rho} / 2\pi \quad (42)$$

where  $\rho$  is the density of the final stage collapse and  $G$  is the constant of gravitation. This gives the total emitted energy as

$$\epsilon \sim \nu^5. \quad (43)$$

This energy could be considerable and could even be observed if it would occur a major collapse within our own galaxy. From the Virgo-cluster a few such events are expected per month with an flux in the order of  $10^{-7}$  J/s which is within reach of near-future detectors.

## 4 Experiments

There exists two main kinds of gravitational wave detectors: Resonance detectors and interferometer detectors. The first one is able to detect signals at their single resonance modes and the last one can detect signals over a whole bandwidth.

### 4.1 Resonance detectors

In the beginning of the 60's, Joseph Weber build the first gravitational wave detector. It was a mechanically isolated cylinder of solid aluminum weighing 1.5 tons. Some piezoelectric strain transducers at its circumference measured the vibrations induced by passing gravitational waves. The bar resonated around the frequency of 1 kHz, which is expected of a gravitational wave from a supernova. The main problem with resonant-bar antennas is their insensitivity. The sensitivity was a distortion of about  $10^{-18}$  meters. However, that is too small to detect gravitational waves from any but the nearest and most violent events. The noise-level of the detector is measured in *strain sensitivity*,  $\tilde{h}_f$  and dimension is  $Hz^{-1/2}$ . For an ordinary resonant detector a noise of  $10^{-22}$  is a typical number for the resonance modes, but better technology will hopefully decrease the noise-level to  $10^{-23}$  in the future.

#### 4.1.1 Construction

The detectors are often build of aluminium. To reduce the thermal noise they are cooled down to the absolute temperature and to reduce the seismic noise they are isolated from vibrations.

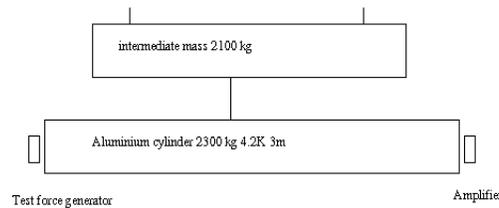


Figure 2:

For an example we have taken the resonance detector, ALLEGRO, in Louisiana. Schematic description in figure 2. It is a cylindric bar cooled down to 4.2K and has a  $\tilde{h}_f = 10^{-21}$  for  $f = 920Hz$  and  $f = 896Hz$ . The detector is not unique, there are several others of similar character.

## 4.2 Interferometer detectors

In 1978, the first gravitational wave detector to be based on a laser interferometry was designed by Forward. It had legs which were two meters long. Five years later, in 1983, a prototype with 40-meter-long legs was built at CalTech. The sensitivity was roughly  $10^{-14}$ . Today the sensitivity is even greater,  $10^{-18}$  being a rather normal value of the sensitivity.

The first generations of large scale-interferometers is under construction; VIRGO in Italy and LIGO in USA. They are expected to start operate in 2002. Two other smaller ones are also being built; GEO300 in Germany and TAMA300 in Japan. Lowfrequency gravitational waves cannot be detected on earth because of the seismic noise. Therefore a space interferometer, LISA, is planned by ESA. One possible target with LISA is to explore the background of gravitational waves from the origin of the universe and probe deeper than we are able to do today. One reason why that is possible is that the graviton interacts very weakly and have a small cross subsection.

### 4.2.1 Construction

The construction of an interferometer detector is simply a Michelson interferometer with very long arms. It is made of optical resonant evacuated cavities with two mirrors, with test masses, at their ends. After going through very long optical path's inside the cavities, two beams of laser light, produced by the same source, are again combined out of phase so that no light gets into the detector. The variation of the optical path length, caused by the changed distance between the mirrors, affected by a gravitational wave, produces a partial phase shift of the beams and, thus, an alteration in the observed luminous intensity which is proportional to the amplitude of the wave. The lenth of the arms of LIGO are 4 km and for VIRGO 3 km and

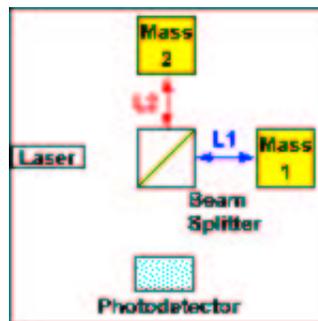


Figure 3:

for the two smaller ones 300 m. Interferometer detectors are wide-band detectors. They can measure from a few Hz to several kHz. On the Earth the band is lowered by the seismic noise. For VIRGO this gives a limit of 2 Hz

and for LIGO 40 Hz. Up to several hundred Hz the thermal noise dominates and after that the laser shot noise will overshadow this effect. The minimum sensitivity for LIGO is about  $10^{-19}$  for 200 Hz and the corresponding for VIRGO is  $10^{-23}$  Hz. All these detectors will be in a network to increase the reliability and the sensitivity. To be able to detect a cosmic gravitational wave width, a single detector with  $\tilde{h}_f(100Hz) < 10^{-24}1/\text{sqrt}Hz$  is needed. Obviously, no single detector can detect a gravitational wave. The second or maybe the third generation of detectors would be able to detect gravitational wave, due to reduced thermal noise thanks to better technology such as better materials in the test masses and increase the laserpower. To reduce the noise, another way is to have a space interferometer, like LISA, which will have a sensitivity of  $\tilde{h}_f = 10^{-23}$  at  $10^{-2}Hz$ .

### 4.3 LISA (Laser Interferometer Space Antenna)

LISA will primarily detect gravitational waves from galactic and extragalactic binary systems and generating gravitational waves from massive black holes in the centre of galaxies.

The three flying space craft will act like a giant Michelson-interferometer, figure 4, measuring the distortion of space caused by passing gravitational waves. Each spacecraft will contain two freely floating test masses. The test

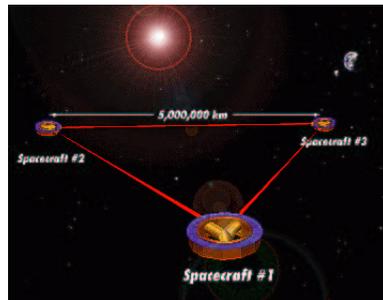


Figure 4:

masses will define optical paths which are 5 million kilometers long, with a 60 degree angle between them. The precession in optical path will be 20 picometers. The spacecraft are planned to be launched in 2008.

The main problem for LISA is to make sure that the distance between the test masses is changed only by gravitational waves, and not by photons and other particles. Therefore the test masses are carefully isolated from any distortion from space. Each of the instrument can act like a beamsplitter or a detector, and the other two will act like the test masses in figure 3. Since they are so far away from each other, they can't just reflect the beam. Instead the light will be transmitted to a new beam.

LISA will observe gravitational waves from massive black holes; bursts

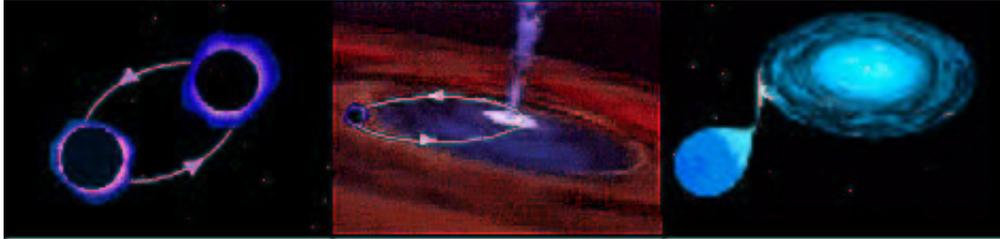


Figure 5: Gravitational waves from objects: Coalescence of black holes (left), black holes orbiting massive black holes (centre), galactic binaries (right).

which come from the terminal stages of binary coalescences and continues waves which come from binaries.

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